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A role of the axial-vector mesons on the photon production in heavy-ion collisions and their relevant decays

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Abstract. A role of the axial-vector mesons, such as K_1 and a_1, on the emitted-photon spectrum in hot hadronic matter is studied through the channels $\pi\rho \to a_1 \to \pi\gamma$ and $K\rho \to K_1 \to K\gamma$. Both channels could be dominant over the region lower than $E_\gamma \sim 0.5$ GeV, while the role of the K_1 meson is diminished in the higher E_γ region. This study is carried out with an $SU_L(3) \otimes SU_R(3)$ effective chiral Lagrangian which includes vector and axial-vector mesons systematically and explains well their hadronic and radiative decays simultaneously.

PACS. 13.25.Jx Decays of other mesons – 13.30.Eg Hadronic decays – 25.75.Dw Particle and resonance production – 25.70.-z Low and intermediate energy heavy-ion reactions

1 Introduction

Recently, the study of dense and hot hadronic matter (HHM) is one of the interesting fields in relativistic heavy-ion physics. One of the main goals for this study is to find a trace of the phase transition between the hadronic and the quark-gluon plasma (QGP) phases. Photons (or dileptons as massive photons) can be used as a reasonable probe to detect this phenomenon because their mean-free paths are much larger than the transverse size of the hot and dense region of the phases. It means that photons escape the region without rescattering, namely, they retain the physical information in each phase.

Therefore, the observation of the QGP signal through the detection of the photons (or dileptons) could be a plausible choice although there remained many discussions [1–6] to be solved yet, in specific, regarding if the emitted photons from the HHM or QGP phases could be discerned.

Yet, to our current knowledge, the HHM and QGP produce energetic photons and massive photons nearly equally at a fixed temperature, 200 MeV, which is the critical temperature for the deconfinement and chiral symmetric phase. Therefore, if any important contributions have been ignored in the emitted-photon spectrum, for example, strange particles or heavy mesons, it would be a very desirable task to investigate those contributions.

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Regarding where the photons are emitted from, we consider the photons produced only through the mesons, *i.e.*, baryon-free matter [3] because baryons are presumed too heavy to be pair produced at the temperature considered in this report. In specific, our interest is aimed to find a role of the axial-vector mesons, which usually participate as intermediate states in the emitted-photon spectrum by the mesons.

Actually, since the beginning of '90, a_1 meson's role, as an intermediate state for the channels $\rho\pi \to \pi\gamma$ and $\pi\pi \to \rho\gamma$, is emphasized [1]. Although ρ and a_1 form a parity doublet, *i.e.*, they are chiral partner just like π and σ, their masses are not degenerated because of the spontaneous breaking of chiral symmetry. In the chiral symmetric phase, however, a_1 becomes to be as important as ρ meson. For instance, Xiong [1] and Song [5] have shown that a_1 meson's contribution, *i.e.*, $\pi\rho \to a_1 \to \pi\gamma$ and $\pi\pi \to a_1 \to \rho\gamma$ could dominate the $\pi\rho \to \pi\gamma$ and $\pi\pi \to \rho\gamma$ channels within the massive Yang-Mills and the gauged linear sigma models, respectively.

Kaon can also scatter with ρ to form a $K_1(1270)$ resonance and decay into kaon and photon finally. Consequently, this K_1 resonance affects largely the kaon mean-free path in HHM, which can be comparable to that of pion [7]. Therefore, the $K\rho \to K_1 \to K\gamma$ channel could become an important source for the photons emitted at this temperature region. But this effect was not taken into account in [7]. Originally, the contribution of the K_1

Table 1. Masses and decay widths of the relevant axial-vector mesons. Xiong calculated only one case of a_1 meson. Haglin used the same Lagrangian as Xiong and extended it to the K_1 case. Radiative decays in both calculations in 1st column were evaluated under the VMD model.

	Xiong [1], Haglin [2]	C. Song [5]	P. Ko [6]	Ours	Experiment [8]
$m_{a_1}(1260)$	1230	1230	1260	1210	1230 ± 40 MeV
$m_{K_1}(1270)$	1273	–	–	1280	1273 ± 7 MeV
$\Gamma_{a_1 \to \rho\pi}$	400	400	328	488	200–600 MeV
$\Gamma_{a_1 \to \pi\gamma}$	1.4	–	0.67	0.688	0.64 ± 0.28 MeV
$\Gamma_{K_1 \to \rho K}$	37.8	–	–	47	57 ± 5 MeV
$\Gamma_{K_1 \to K\gamma}$	1.5	–	–	0.350	

meson to a radiative decay in a hadron gas was studied by Haglin [2], but with a simple phenomenological Lagrangian for hadron decays and a simple-minded vector-meson dominance (VMD) model for radiative decays. Moreover, their decay widths are overestimated when they are compared to the experimental data (see table 1).

In case of electro-magnetic interactions of the hadron at energy below 1 GeV, the vector meson plays an important role. The VMD model has been proved remarkably successful in the description of electro-magnetic form factors and decays of the hadron. But it is a phenomenological approach, *i.e.*, it is a kind of a hypothesis. Moreover, the model sometimes turned out to need more quantitative developments to account for more refined experimental data. In consequence, one needs models beyond the VMD model.

Many approaches, such as a hidden gauge symmetry approach (HGS) [9,10], a massive Yang-Mills approach (MYM) [11], and so on, have been developed to include the vector meson in a fundamental manner. For instance, the ρ meson is treated as a gauge boson in HGS or an explicit degree of freedom in MYM approach. All the models can be shown to be equivalent [12] by taking higher-order terms into account, redefining suitable fields and adjusting parameters. In some approach, such as the $O(p^4)$-order expansion of the chiral perturbation theory (ChPT), the $\mathcal{L}_2$ Lagrangian gives higher-orders loop contributions as well known [13], which helps a good phenomenological description beyond the VMD. However, such a simple addition of higher-order terms is not a convenient method for those calculations.

In our previous paper [14], we have proposed an effective $SU(3)_L \otimes SU(3)_R$ chiral Lagrangian for the description of vector and axial-vector mesons by considering all the relevant symmetries and the low-energy constraints from the ChPT. Moreover, our effective Lagrangian theory, which is aimed for a large-energy process, uses the $O(p^2)$ expansion because most of the higher-order contributions in other approaches are incorporated by a single change in the kinetic terms of the vector field with only one parameter in our model. In specific, we would like to remind that the chiral symmetry is the most important symmetry ought to be retained in the study of HHM.

In this paper, K_1 as well as a_1 axial-vector mesons' contributions to the photon spectrum in HHM are calculated in our effective Lagrangian and shown to be dominant channels in this spectrum. Our recipe reproduces well axial-vector mesons' decays, such as $a_1 \to \rho\pi$, $a_1 \to \pi\gamma$, $K_1 \to K\rho$ and $K_1 \to K\gamma$, in a consistent manner. This successful description was resulted from the systematic extension of the $SU(2)$ group representation to that of $SU(3)$ [14].

This paper is organized as follows. In sect. 2, the Lagrangian used here is briefly reviewed. Relevant decays of the axial-vector mesons are calculated in sect. 3 in the framework of this Lagrangian and compared to other calculations and experimental data available until now. The photon spectra via $\pi\rho \to a_1 \to \pi\gamma$ and $K\rho \to K_1 \to K\gamma$ channels are investigated in sect. 4. A brief summary is done in sect. 5.

2 Lagrangian

In the previous paper [14], experimental data relevant to pion and kaon, such as form factors and charge radii, were reproduced with only mass terms and kinetic terms of spin-1 meson fields. The spin-1 mesons are introduced in the non-linear realization of chiral symmetry, with which it is easy to check consistency with the ChPT. Since full reviews concerning other effective theories and their relationships among other approaches can be found in other papers [11,12], we skip them here.

Our Lagrangian consists of a pseudoscalar-meson sector $\mathcal{L}(\pi)$, a spin-1 vector and axial-vector-meson sector $\mathcal{L}(V,A)$, and a term of interactions with scalar particles $\mathcal{L}_S$, which comes from a mass splitting in the $SU(3)$ extension of the previous $SU(2)$ Lagrangian [15], *i.e.*,

$$\mathcal{L} = \mathcal{L}(\pi) + \mathcal{L}(V,A) + \mathcal{L}_S \ . \tag{1}$$

The Lagrangian for the pseudoscalar-meson sector, which is a leading Lagrangian in ChPT, is given as

$$\mathcal{L}(\pi) = \frac{f^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle + \frac{f^2}{4}\langle U^\dagger \chi + \chi^\dagger U\rangle \ , \tag{2}$$

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \ , \tag{3}$$

where brackets denote a trace in flavor space, f is a pseudoscalar-meson decay constant, the chiral field is denoted as $U = \exp(i2\pi/f)$ with $\pi = T^a\pi^a$, $T^a = \lambda^a/2(a = 1,2,...8)$. External gauge fields are introduced via v_μ and a_μ. The χ is defined by $\chi = 2B_0(\mathcal{S} + i\mathcal{P})$. B_0 is a constant related to the scalar-quark condensation. Explicit chiral symmetry breaking due to current quark masses can be introduced by treating those masses as if they were uniform external scalar fields S [12].

The non-linear realization of chiral symmetry is expressed in terms of $u = \sqrt{U}$ and $h = h(u, g_R, g_L)$ defined from $u \to g_R u h^\dagger = h u g_L^\dagger$. In this realization, we naturally have the following covariant quantities:

$$\begin{aligned} i\Gamma_\mu &= \frac{i}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) + \frac{1}{2}u^\dagger(v_\mu + a_\mu)u \\ &\quad + \frac{1}{2}u(v_\mu - a_\mu)u^\dagger \ , \\ i\Delta_\mu &= \frac{i}{2}(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) + \frac{1}{2}u^\dagger(v_\mu + a_\mu)u \\ &\quad - \frac{1}{2}u(v_\mu - a_\mu)u^\dagger \ , \\ \chi_+ &= u^\dagger \chi u^\dagger + u \chi u \ , \end{aligned} \tag{4}$$

whose transformations are carried out in terms of h, *i.e.*, $\Gamma_\mu \to h\Gamma_\mu h^\dagger - \partial_\mu h \cdot h^\dagger$, $\Delta_\mu \to h\Delta_\mu h^\dagger$, and $\chi_+ \to h\chi_+ h^\dagger$. With these quantities, the Lagrangian in eq. (2) can be rewritten as

$$\mathcal{L}(\pi) = f^2 \langle i\Delta_\mu i\Delta^\mu \rangle + \frac{f^2}{4}\langle \chi_+ \rangle. \tag{5}$$

As for the massive spin-1 mesons, we include only mass and kinetic terms [15]

$$\begin{aligned} \mathcal{L}(V,A) = {f_V}^2 g^2 \left\langle \left(V_\mu - \frac{i\Gamma_\mu}{g} \right)^2 \right\rangle + {f_A}^2 g^2 \left\langle \left(A_\mu - \frac{ir\Delta_\mu}{g} \right)^2 \right\rangle \\ - \frac{1}{2}\left\langle \left({}^G V_{\mu\nu}\right)^2 \right\rangle - \frac{1}{2}\left\langle (A_{\mu\nu})^2 \right\rangle \ , \end{aligned} \tag{6}$$

with

$$\begin{aligned} {}^G V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] - iG[A_\mu, A_\nu] \ , \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[V_\mu, A_\nu] - ig[A_\mu, V_\nu] \ , \end{aligned} \tag{7}$$

where $V_\mu = T^a V_\mu^a (A_\mu = T^a A_\mu^a)$ denotes the spin-1 vector (axial-vector) meson field and g is a $V\pi\pi$ coupling constant. The parameter r can be determined from the KSRF relation ($m_V^2 \sim 2g^2 f^2$) and Weinberg's two sum rules [16] ($m_V^2 - r^2 m_A^2 \sim g^2 f^2$ and $m_V^2 - r m_A^2 \sim 0$). The chiral transformation rules of spin-1 fields are expressed in terms of h

$$V_\mu \to h V_\mu h^\dagger - \frac{i}{g}\partial_\mu h \cdot h^\dagger \ , \qquad A_\mu \to h A_\mu h^\dagger. \tag{8}$$

Note that we have introduced a new form of ${}^G V_{\mu\nu}$. The chiral symmetry is preserved for any value of G at the chiral limit in ${}^G V_{\mu\nu}$, so that the value of G cannot be determined from the chiral symmetry. If G is equal to g as in the HGS approach [9,10], the result may reproduce experimental data by explicitly including other higher-order terms.

The $\mathcal{L}_S$ term includes effects coming from a mass splitting of strange and non-strange particles in terms of interaction Lagrangians between scalar particles and other mesons (pseudoscalar, vector and axial-vector mesons). Detailed derivation was presented in ref. [14]. The resulting Lagrangian is given as

$$\begin{aligned} \mathcal{L}_S &\sim -\frac{1}{2}\left(\frac{s_m}{f}\right)^2 (\tilde{M})_a^2 (\pi^a)^2 + \frac{1}{2}s_m M_a j^a, \\ j &= s_d (i\Delta_\mu)^2 + s_V (gV_\mu - i\Gamma_\mu)^2 + s_A (gA_\mu - ir\Delta_\mu)^2 \\ &\quad + s_r \{ i\Delta^\mu, gA_\mu - ir\Delta_\mu \} \ , \end{aligned} \tag{9}$$

where $\tilde{M}_a^2 = \frac{1}{6}(2B_0\alpha)^2 \delta_{8a} + M_a^2$. j stands for the interaction, on which s_d, s_V, s_A, s_r, and s_m are free parameters fitted to determine the masses of related mesons (see eq. (14)). Current quark mass matrix M_a (we assume that masses of u and d quarks are equal) is given as

$$\begin{aligned} M_a &= 2B_0 \left(\frac{D_a}{2\sqrt{3}}\alpha + \beta \right), \\ \frac{D_a}{2\sqrt{3}}\alpha + \beta &= \begin{cases} \overline{m}\,, & \text{for } a = 1,2,3\,, \\ \frac{1}{2}(\overline{m} + m_s)\,, & \text{for } a = 4,5,6,7\,, \\ \frac{1}{3}(\overline{m} + 2m_s)\,, & \text{for } a = 8\ . \end{cases} \end{aligned} \tag{10}$$

For the mixing between axial-vector mesons and pion fields, we define $A_\mu^{'}$ as

$$A_\mu^a = A_\mu^{\prime a} - \frac{r}{gf}\partial_\mu \pi^a + \frac{r}{gf} f_{abc} \pi^b B_\mu^c \ , \tag{11}$$

where B_μ denotes a photon field and $Q = T^3 + \frac{Y}{2}$. This field redefinition of eq. (11), which differs from our previous one, leads to an explicitly manifest gauge invariance of the amplitude relevant to the radiative decay of the axial-vector meson. But it does not give any differences in V-π-γ reaction in our previous investigation, and describes correctly A-V-π and A-γ-π reactions.

As for the mixing between vector mesons and pion fields, we also define $V_\mu^{'}$ as

$$V_\mu^a = V_\mu^{\prime a} - \frac{Gr^2}{2g^2 f^2} f_{abc} \pi^b \partial_\mu \pi^c. \tag{12}$$

Besides the above mixings, we introduced the ρ-ω mixing and the mixing between the vector meson and photon field to avoid unphysical coupling related to the photon field. This mixing term allows a direct coupling of the photon to the pion and leads to a small, but very important, deviation from the conventional VMD model, so that, it explains experimental data well rather than the VMD model prediction (see Figure 2 and 3 in ref. [14]).

The Lagrangian is, then, finally summarized as

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}m_{Va}^2 V_\mu V^\mu + \frac{1}{2}m_{Aa}^2 A_\mu A^\mu \\
& + \frac{m_{Va}^2}{2gf_a^2}\left(1 - \frac{Gr^2}{g}\right) f_{abc} V_\mu^a \pi^b \partial^\mu \pi^c \\
& + eQ f_{abc} B_\mu^a \pi^b \partial^\mu \pi^c \\
& - \frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 - \frac{1}{4}\left(\frac{e}{g}\right)^2 Q^2 (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \\
& - \frac{e}{2g} Q (\partial_\mu V_\nu - \partial_\nu V_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) \\
& + \frac{1}{2}\frac{Gr}{gf} f_{abc} (\partial_\mu V_\nu^a - \partial_\nu V_\mu^a)(A^{\mu b}\partial^\nu \pi^c + \partial^\mu \pi^b A^{\nu c}) \\
& + \frac{1}{2}\frac{Gr}{gf}\left(\frac{e}{g}\right) Q f_{abc} (\partial_\mu B_\nu^a - \partial_\nu B_\mu^a) \\
& \times (A^{\mu b}\partial^\nu \pi^c + \partial^\mu \pi^b A^{\nu c}) \\
& - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
& + \frac{1}{2}\frac{r}{f} f_{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(V^{\mu b}\partial^\nu \pi^c + \partial^\mu \pi^b V^{\nu c}) \\
& - \frac{1}{2}\frac{r}{f}\frac{e}{g} Q f_{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)\pi^b (\partial^\mu B^{\nu c} - \partial^\nu B^{\mu c}) \\
& - \frac{1}{2}m_{\pi a}^2 \pi^a \pi^a + \frac{1}{2}\partial_\mu \pi^a \partial^\mu \pi^a, \qquad (13)
\end{aligned}$$

where $m_{Va}^2 = g^2(f_V^2 + s_m s_v M_a)$, $m_{Aa}^2 = g^2(f_A^2 + s_m s_a M_a)$. V_μ and A_μ fields stand for redefined fields V_μ' and $A_\mu{}'$. In order to determine the pseudoscalar-meson mass and decay constants, we exploit the following covariant quantities:

$$m_{\pi a}^2 = (M_a + \left(\frac{s_m}{f}\right)^2 \tilde{M_a}^2)/Z_{\pi a}^2\,, \qquad f_a = Z_{\pi a} f\,,$$
$$\text{with} \quad Z_{\pi a}^2 = \left(1 + s_m s_d \frac{M_a}{f^2}\right)\,. \qquad (14)$$

The relation of our chiral effective Lagrangian to the other Lagrangians was discussed in refs. [14,12]. The 9th and 12th terms, and the 8th and 11th terms in the above Lagrangian, which were omitted in our previous Lagrangian, corresponds to the A-γ-π and A-V-π interactions, respectively. Detailed discussions concerning the application of this Lagrangian to decay modes are carried out in the next section.

3 Axial-vector meson decay

Our Lagrangian seems to be a very complicated Lagrangian, at a glance. But we would like to make a stress on its systematic and effective properties, and its consistency with other chiral Lagrangians, as explained already.

Here it would be a pedagogical step to explain the reason why the axial-vector meson decay should be revisited in this Lagrangian. In the 2nd column in table 1 we showed the results of Haglin and Xiong *et al.* In specific, they did not obtain reasonable values for radiative decays, even though the results for hadron decays seem to be consistent with the experimental data. This is the limit of the VMD model used in their calculations. A simply naive picture for electro-magnetic interactions, just like the VMD model, cannot reproduce more refined data any more. This is true of electro-magnetic form factors of the pion and kaon. Therefore, one needs to develop more fundamental approaches beyond the VMD model. For instance, the calculation of Ko *et al.* in the 4th column is such a case. But they considered only the case of a_1 meson. Therefore, K_1 as well as a_1 meson should be considered again in the framework beyond VMD model. This is a motivation for our work.

One more point is that we have to explain both a_1 and K_1 data simultaneously. As emphasized in the papers of Haglin [7], the kaon mean free path in HHM is affected largely by the K_1 resonance, and as a result it is comparable to that of the pion. Therefore, kaon could be also an important source for the emitted photons. It means that one has to explain both mesons' properties simultaneously in order to investigate roles of the axial-vector mesons in HHM. But the K_1 is a strange meson, so that a chiral $SU(2)$ Lagrangian should be extended to $SU(3)$ by including the chiral-symmetry breaking effect due to a mass splitting. Our Lagrangian, in this sense, is well suited to these purposes.

3.1 Hadron decay

The Lagrangian for the A-V-π reaction is given as

$$\begin{aligned}
\mathcal{L}_{AV\pi} = & \frac{1}{2}\frac{Gr}{gf} f_{abc}(\partial_\mu V_\nu^a - \partial_\nu V_\mu^a)(A^{\mu b}\partial^\nu \pi^c + \partial^\mu \pi^b A^{\nu c}) \\
& + \frac{1}{2}\frac{r}{f} f_{abc}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(V^{\mu b}\partial^\nu \pi^c + \partial^\mu \pi^b V^{\nu c}).
\end{aligned} \qquad (15)$$

The 1st term corresponds to the Lagrangian used by Xiong [1] and Haglin [2]. The 2nd term comes from the $\partial_\mu \pi$ in eq. (11), which avoids a direct coupling of the pion to the axial-vector mesons. From the above Lagrangian, the partial decay width of the a_1 meson to $\rho\pi$, $a_1(p,\ \epsilon_A) \to \rho(k,\ \epsilon_V)\pi(q)$, is calculated as follows:

$$\begin{aligned}
\Gamma_{a_1^\pm \to \rho\pi} &= \Gamma_{a_1^\pm \to \rho^\pm \pi^0} + \Gamma_{a_1^\pm \to \rho^0 \pi^\pm}\,, \\
\Gamma_{a_1^\pm \to \rho^\pm \pi^0 (a_1^\pm \to \rho^0 \pi^\pm)} &= \frac{1}{24\pi}\frac{|\vec{q}\,|}{m_a^2}|\mathcal{M}_{a_1^\pm \to \rho^\pm \pi^0 (a_1^\pm \to \rho^0 \pi^\pm)}|^2\,,
\end{aligned} \qquad (16)$$

where $\vec{q}$ is an incoming pion momentum in the a_1 rest frame given as $\vec{q}^{\,2} = \frac{(m_{a_1}^2 + m_\pi^2 - m_\rho^2)^2}{4m_{a_1}^2} - m_\pi^2$. Invariant amplitudes $\mathcal{M}$ and $|\mathcal{M}|^2$ for the decay are presented as

$$\mathcal{M}_{a_1\to\rho\pi} = f_1\epsilon_{V\nu}((q\cdot k)g^\nu_\mu - k_\mu q^\nu)\epsilon^\mu_A + f_2\epsilon_{V\nu}((p\cdot q)g^\nu_\mu - p_\mu q^\nu)\epsilon^\mu_A\ , \quad (17)$$

$$|\mathcal{M}_{a_1^\pm\to\rho^0\pi^\pm}|^2 = |\mathcal{M}_{a_1^\pm\to\rho^\pm\pi^0}|^2 = 4[f_1(2(k\cdot q)^2 + m^2_\rho(m^2_\pi + \vec{q}^{\ 2})) + f_2(2(p\cdot q)^2 + (q\cdot k)^2) + 6f_1f_2(p\cdot q)(k\cdot q)]\,. \quad (18)$$

Coupling constants f_1, f_2 are defined as $f_1 = \frac{1}{2}\frac{Gr}{gf}$, $f_2 = \frac{1}{2}\frac{r}{f}$. p, q and k stand for momenta of the axial-vector meson, the vector meson, and the pion, respectively. The partial decay width of the K_1 meson to ρK, $K_1(p,\ \epsilon_A) \to \rho(k,\ \epsilon_V)K(q)$, is calculated in the same manner,

$$\Gamma_{K_1^\pm\to\rho K} = \Gamma_{K_1^\pm\to\rho^\pm K^0} + \Gamma_{K_1^\pm\to\rho^0K^\pm}\ ,$$

$$\Gamma_{K_1^\pm\to\rho K} = \frac{1}{24\pi}\frac{|\ \vec{q}\ |}{m^2_{K_1}}|\mathcal{M}_{K_1^\pm\to\rho K}|^2\ , \quad (19)$$

where the invariant amplitudes $\mathcal{M}$ and $|\mathcal{M}|^2$ are written as

$$\mathcal{M}_{K_1\to\rho K} = f_{K_1}\epsilon_{V\nu}((q\cdot k)g^\nu_\mu - k_\mu q^\nu)\epsilon^\mu_A + f_{K_2}\epsilon_{V\nu}((p\cdot q)g^\nu_\mu - p_\mu q^\nu)\epsilon^\mu_A\ , \quad (20)$$

$$|\mathcal{M}_{K_1^\pm\to\rho^0K^\pm}|^2 = [f_{K_1}(2(k\cdot q)^2 + m^2_\rho(m^2_{K^\pm} + \vec{q}^{\ 2})) + f_{K2}(2(p\cdot q)^2 + (q\cdot k)^2) + 6f_{K_1}f_{K2}(p\cdot q)(k\cdot q)]\,,$$

$$|\mathcal{M}_{K_1^\pm\to\rho^\pm K^0}|^2 = 2[f_{K_1}(2(k\cdot q)^2 + m^2_\rho(m^2_{K^0} + \vec{q}^{\ 2})) + f_{K2}(2(p\cdot q)^2 + (q\cdot k)^2) + 6f_{K_1}f_{K2}(p\cdot q)(k\cdot q)]\,. \quad (21)$$

Here f_{K_1}, f_{K_2} are defined as $f_{K_1} = \frac{1}{2}\frac{Gr}{gf_K}$, $f_{K_2} = \frac{1}{2}\frac{r}{f_K}$, with a kaon decay constant f_K. In this case, the incoming momentum $\vec{q}$ is $\vec{q}^{\ 2} = \frac{(m^2_{K_1}+m^2_K-m^2_\rho)^2}{4m^2_{K_1}} - m^2_K$, and p, q, k are the K_1 meson, the ρ meson, the kaon momentum, respectively.

3.2 Radiative decay

On the other hand, the Lagrangian for the A-γ-π reaction is obtained from eq. (13) as follows:

$$\mathcal{L}_{A\gamma\pi} = \frac{1}{2}\frac{Gr}{gf}\left(\frac{e}{g}\right)Qf_{abc}(\partial_\mu B^a_\nu - \partial_\nu B^a_\mu) \times(A^{\mu b}\partial^\nu\pi^c + \partial^\mu\pi^b A^{\nu c}) - \frac{1}{2}\frac{r}{f}\left(\frac{e}{g}\right)Qf_{abc} \times(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\pi^b(\partial^\mu B^{\nu c} - \partial^\nu B^{\mu c})\ . \quad (22)$$

The 1st term resembles the Lagrangian of Xiong [1], but in our Lagrangian, the 2nd term appears, which is originated from the 3rd term in eq. (11) and makes the relevant amplitudes gauge invariant.

Partial decay widths of the a_1 meson to $\gamma\pi$ and K_1 meson to γK, $a_1(p,\ \epsilon_A) \to \gamma(k,\ \epsilon_\gamma)\pi(q)$ and $K_1(p,\ \epsilon_A) \to \gamma(k,\ \epsilon_\gamma)K(q)$, are expressed as

$$\Gamma_{a_1^\pm\to\gamma\pi^\pm} = \frac{1}{24\pi}\frac{|\vec{q}|}{m^2_{a_1}}|\mathcal{M}_{a_1^\pm\to\gamma\pi^\pm}|^2\ ,$$

$$\Gamma_{K_1^\pm\to\gamma K^\pm} = \frac{1}{24\pi}\frac{|\vec{q}|}{m^2_{K_1}}|\mathcal{M}_{K_1^\pm\to\gamma K^\pm}|^2\ , \quad (23)$$

where the invariant amplitudes $\mathcal{M}$ and $|\mathcal{M}|^2$ are given as

$$\mathcal{M}_{a_1\to\pi\gamma} = h_1\epsilon_{\gamma\nu}((q\cdot k)g^\nu_\mu - k_\mu q^\nu)\epsilon^\mu_A - h_2\epsilon_{\gamma\nu}((p\cdot k)g^\nu_\mu - k_\mu p^\nu)\epsilon^\mu_A\ , \quad (24)$$

$$|\mathcal{M}_{a_1^\pm\to\gamma\pi^\pm(K_1^\pm\to\gamma K^\pm)}|^2 = 4[2h_1(k\cdot q)^2 + 2h_2(k\cdot p)^2 - 4h_1h_2(k\cdot q)(k\cdot p)]\ . \quad (25)$$

Here $\vec{q}^{\ 2} = \left(\frac{m^2_{a_1}-m^3_\pi}{2m_{a_1}}\right)^2$ and $h_1 = \frac{e}{g}f_1, h_2 = \frac{e}{g}f_2$, with f_1 and f_2 defined in the previous subsection. p, q and k are the axial-vector meson, the vector meson, the photon momentum, respectively. It is easy to check that the above amplitude is fully gauge invariant.

By exploiting the above equations, we determine the values of G, r and g by fitting our numerical predictions of eqs. (16), (19) and (23) to the experimental data. Our results for the decays and experimental data [8] are tabulated in table 1 with the comparison to other cases. Our results show quite reasonable values in both hadronic and radiative decays for a_1 and K_1 mesons. In specific, in case of radiative decays, 2 or 3 times smaller values than those of VMD model are obtained. The masses of axial-vector meson used here are $a_1(1260) = 1210\,\text{MeV}$, $K_1(1270) = 1280\,\text{MeV}$, respectively.

4 Photon production rate

In this section, we show a brief formalism regarding the emitted-photon spectrum in a hadronic gas. Since the mean free paths of emitted photons are much larger than the transverse size of HHM, we assume that the photons emerged from the matter without final-state scattering. The thermal rate of photons with energy E and momentum $\mathbf{p}$ is related to the imaginary of the retarded photon self-energy $\text{Im}\Pi^{R,\mu}_\mu$ with a thermal weighting [17]

$$E\frac{\text{d}R}{\text{d}^3p} = \frac{-2}{(2\pi)^3}\text{Im}\Pi^{R,\mu}_\mu\frac{1}{e^{E/T}-1}, \quad (26)$$

where R is the number of the produced photons per unit time and unit volume. It is valid to all orders in the strong interactions, but only to order e^2 in the electro-magnetic interactions. In order to obtain the photon emission rate, we need to evaluate the imaginary part of the photon self-energy. The Cutkosky rules at finite temperature give a systematic procedure to calculate the imaginary part of

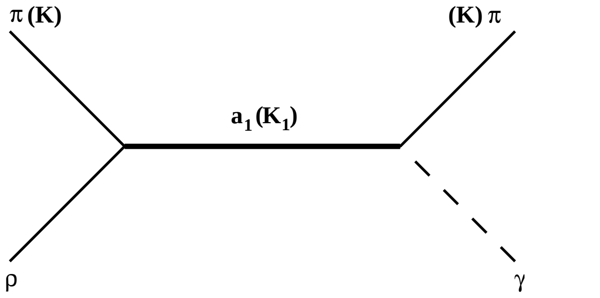

Fig. 1. Feynman diagram of $\pi\rho \to \pi\gamma(K\rho \to K\gamma)$ through the $a_1(K_1)$ resonance.

a Feynman diagram. If the photon self-energy is approximated by carrying out a loop expansion to some finite order, eq. (26) becomes equivalent to the relativistic kinetic theory.

In principle, including the axial-vector mesons as intermediate states can add other channel processes in photon production. However, we consider only an s-channel contribution as in fig. 1, because contributions of other channels are known to be small compared to that of the s-channel in the hadronic gas at $T = 100$–200 MeV [1,6].

For a reaction of the mesons, $1 + 2 \to 3 + \gamma$, the photon production rate with temperature T [18] is given by

$$E\frac{\mathrm{d}R}{\mathrm{d}^3p} = \frac{\mathcal{N}}{16(2\pi)^7 E}\int_{(m_1+m_2)^2}^{\infty}\mathrm{d}s\int_{t_{\min}}^{t_{\max}}\mathrm{d}t\,|\mathcal{M}|^2\int\mathrm{d}E_1 \times\int\mathrm{d}E_2\frac{f(E_1)\,f(E_2)\,[1+f(E_3)]}{\sqrt{aE_2^2+2bE_2+c}}, \quad (27)$$

where

$$a = -(s+t-m_2^2-m_3^2)^2\ ,$$
$$b = E_1(s+t-m_2^2-m_3^2)(m_2^2-t) +E_\gamma[(s+t-m_2^2-m_3^2)(s-m_1^2-m_2^2) -2m_1^2(m_2^2-t)]\ ,$$
$$c = -E_1^2(m_2^2-t)^2 - 2E_1E_\gamma[2m_2^2(s+t-m_2^2-m_3^2) -(m_2^2-t)(s-m_1^2-m_2^2)] - E_\gamma^2[(s-m_1^2-m_2^2)^2 -4m_1^2m_2^2] - (s+t-m_2^2-m_3^2)(m_2^2-t) \times(s-m_1^2-m_2^2) + m_2^2(s+t-m_2^2-m_3^2)^2 +m_1^2(m_2^2-t)^2\ ,$$

$$E_{1\min} = \frac{(s+t-m_2^2-m_3^2)}{4E_\gamma} + \frac{E_\gamma m_1^2}{s+t-m_2^2-m_3^2}\ ,$$
$$E_{2\min} = \frac{E_\gamma m_2^2}{m_2^2-t} + \frac{m_2^2-t}{4E_\gamma}\ ,$$
$$E_{2\max} = -\frac{b}{a} + \frac{\sqrt{b^2-ac}}{a}\ ,$$

where $\mathcal{N}$ is the overall degeneracy of the particle 1 and 2, $\mathcal{M}$ is the invariant amplitude of the reactions considered in this paper (summed over final states and averaged over initial states), and s, t, u are usual Mandelstam variables. In the above equation, indices 1, 2, 3 and γ mean the incident pion, the incident ρ meson, the outgoing pion and the outgoing photon, respectively. They are allowed to vary in a whole range to take off-shell properties of the relevant particles into account. $f(E) = \frac{1}{(e^{E/T}-1)}$ is the Bose-Einstein distribution function.

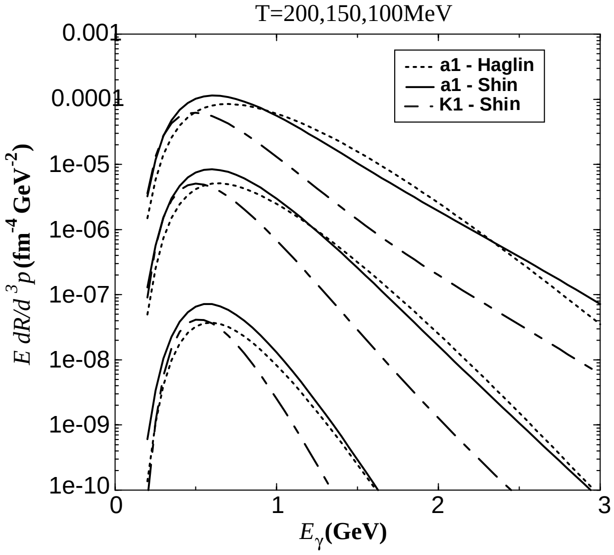

Fig. 2. Photon production rate at $T = 100$–200 MeV. From the uppermost $T = 200,\ 150,\ 100$, respectively.

The invariant amplitude $\mathcal{M}$ for the diagram in fig. 1 is calculated as

$$\mathcal{M} = 2\epsilon_{V\alpha}[f_1((q\cdot k)g_\mu^\alpha - k_\mu q^\alpha) +f_2((p\cdot q)g_\mu^\alpha - p_\mu q^\alpha)]D^{\mu\beta} \times 2\epsilon_{\gamma\nu}[h_1((i\cdot j)g_\beta^\nu - j_\beta i^\nu) -h_2((p\cdot j)g_\beta^\nu - j_\beta p^\nu)]\ , \quad (28)$$

where k and q are the momenta of the incoming ρ and π mesons, i, j are the outgoing photon and π momentum and p is the axial-vector meson's momentum, respectively. $D^{\mu\beta}$ is a propagator for the axial-vector meson

$$D^{\mu\beta} = (g^{\mu\beta} - p^\mu p^\beta)\frac{1}{p^2 - m^2_{a_1(K_1)} - \mathrm{Im}_{a_1(K_1)}\Gamma_{a_1(K_1)}}\ . \quad (29)$$

The processes we are going to study are $\pi\rho \to a_1 \to \pi\gamma$ and $K\rho \to K_1 \to K\gamma$ in fig. 1. Using the approximation by W. Greiner [19], we numerically compute a three-dimensional integral in eq. (27) and get photon spectra at various temperatures. Results are shown in fig. 2. a_1 and K_1 resonance's contributions are presented as solid and dot-dashed curves, respectively. The spectra are calculated at three different temperatures, $T = 200, 150$, and 100 MeV from the uppermost, respectively.

To investigate a dependence of the spectra on the given Lagrangian, previous results [2] for a_1 resonance, which are reproduced by switching off the 2nd term in eq. (15), are shown as dotted curves. Likewise to the discussion in ref. [6], there does not appear discernible differences between dotted and solid curves, although these different Lagrangians give rise to nearly 2 times different results for the radiative decays. It implies that other physical factors in HHM, such as relativistic kinematics and thermodynamics, are also important ingredients for this spectra. Another observation for the case of the a_1 meson is that maximum values in the spectra are increased within 10% at each temperature. It is also noticeable that the peak positions of E_γ in the spectra are shifted a little bit backward compared to the previous predictions.

The K_1 contribution turned out to be smaller in high E_γ region at least one order than that of a_1. But at low E_γ region, it shows a comparable behavior to that of a_1. Therefore, in the region lower than $E_\gamma \sim 0.5$ GeV, both a_1 and K_1 axial-vector mesons show competitive roles in these spectra. But the K_1 meson's role in the high E_γ region above 0.5 GeV is much smaller than that of the a_1 meson.

5 Conclusion

Based on our previous $SU_L(3) \otimes SU_R(3)$ chiral Lagrangian [14], K_1 and a_1 meson's contributions through $K\rho \to K_1 \to K\gamma$ and $\pi\rho \to a_1 \to \pi\gamma$ channels are investigated for the emitted-photon spectrum in a hot hadronic matter. Before the calculation, the relevant decay widths are quite well reproduced within experimental errors. In specific, radiative and hadronic decays for the axial-vector mesons, a_1 and K_1, are shown to be consistently explained in our Lagrangian. Our results for the emitted-photon spectra in a hot and dense matter do not show any drastic change compared to the previous results. But they can pin down the ambiguity possible on radiative decays by comparing our model Lagrangian which is beyond the VMD model and the old-fashioned VMD model. Our emitted-photon spectra show that K_1 and a_1 mesons could dominate over the region lower than $E_\gamma \sim 0.5$ GeV, while the role of the K_1 meson is diminished in the higher E_γ region.

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